[M]athematics... is the easiest of sciences, a fact which is obvious in that no one's brain rejects it....

Roger Bacon (c. 1214-c. 1294), English philosopher and scientist

Stand firm in your refusal to remain conscious during algebra. In real life, I assure you, there is no such thing as algebra.

Fran Lebowitz (b. 1951), American journalist

You may agree with Roger Bacon that mathematics is the easiest of sciences, but many beginning chemistry students would not. Because they have found mathematics challenging, they wish it were not so important for learning chemistry—or for answering so many of the questions that arise in everyday life. They can better relate to Fran Lebowitz’s advice in the second quotation. If you are one of the latter group, it will please you to know that even though there is some algebra in chemistry, this chapter teaches a technique for doing chemical calculations (and many other calculations) without it. Although this technique has several common names, it is called unit analysis in this text. You will be using it throughout the rest of this book, in future chemistry and science courses, and in fact, any time you want to calculate the number of nails you need to build a fence, or the number of rolls of paper you require to cover the kitchen shelves.

Review Skills

The presentation of information in this chapter assumes that you can already perform the tasks listed below. You can test your readiness to proceed by answering the Review Questions at the end of the chapter. This might also be a good time to read the Chapter Objectives, which precede the Review Questions.

- List the metric units without prefixes and the corresponding abbreviations for length (meter, m), mass (gram, g), and volume (liter, L). (Section 1.4)
- State the numbers or fractions represented by the following metric prefixes, and write their abbreviations: giga, mega, kilo, centi, milli, micro, nano, and pico. (Section 1.4)
- Given a metric unit, write its abbreviation; given an abbreviation, write the full name of the unit. (Section 1.4)
- Describe the relationships between the metric units that do not have prefixes (such as meter, gram, and liter) and units derived from them by the addition of prefixes—for example, 1 km = 10^3 m. (Section 1.4)

- Describe the Celsius, Fahrenheit, and Kelvin scales used to report temperature values. (See Section 1.4)
- Given a value derived from a measurement, identify the range of possible values it represents, on the basis of the assumption that its uncertainty is ±1 in the last position reported. (For example, 8.0 mL says the value could be from 7.9 mL to 8.1 mL.) (Section 1.5)
2.1 Unit Analysis

Many of the questions asked in chemistry and in everyday life can be answered by converting one unit of measure into another. For example, suppose you are taking care of your nephew for the weekend, and he breaks his arm. The doctor sets the arm, puts it in a cast, and prescribes an analgesic to help control the pain. Back at home, after filling the prescription, you realize that the label calls for 2 teaspoons of medicine every six hours, but the measuring device that the pharmacy gave you is calibrated in milliliters. You can't find a measuring teaspoon in your kitchen, so you've got to figure out how many milliliters are equivalent to 2 tsp. It's been a rough day, there's a crying boy on the couch, and you're really tired. Is there a technique for doing the necessary calculation that is simple and reliable? Unit analysis to the rescue!

The main purpose of this chapter is to show you how to make many different types of unit conversions, such as the one required above. You will find that the stepwise thought process associated with the procedure called unit analysis\(^1\) not only guides you in figuring out how to set up unit conversion problems but also gives you confidence that your answers are correct.

An Overview of the General Procedure

\textit{In every affair, consider what precedes and follows, and then undertake it.}

Epictetus (c. 55-c. 135) Greek Philosopher

You will see many different types of unit conversions in this chapter, but they can all be worked using the same general procedure. To illustrate the process, we will convert 2 teaspoons to milliliters and solve the problem of how much medicine to give the little boy described above.

The first step in the procedure is to identify the unit for the value we want to calculate. We write this on the left side of an equals sign. Next, we identify the value that we will convert into the desired value, and we write it on the right side of the equals sign. (Remember that a value constitutes both a number and a unit.) We want to know how many milliliters are equivalent to 2 tsp. We express this question as

\[ ? \text{ mL} = 2 \text{ tsp} \]

Next, we multiply by one or more conversion factors that enable us to cancel the unwanted units and generate the desired units. A conversion factor is a ratio that describes the relationship between two units. To create a conversion factor for converting teaspoons to milliliters we can look in any modern cookbook (check its index under “metric conversions”) and discover that the relationship between teaspoons and milliliters is

\[ 1 \text{ tsp} = 5 \text{ mL} \]

\(^1\)Unit analysis has other names, including the factor-label method, the conversion factor method, and dimensional analysis.
This relationship can be used to produce two ratios, or conversion factors:

\[
\frac{5 \text{ mL}}{1 \text{ tsp}} \quad \text{or} \quad \frac{1 \text{ tsp}}{5 \text{ mL}}
\]

The first of these can be used to convert teaspoons to milliliters, and the second can be used to convert milliliters to teaspoons.

The final step in the procedure is to multiply the known unit (2 tsp) by the proper conversion factor, the one that converts teaspoons to milliliters.

\[
\text{Desired unit} \quad \text{Conversion factor} \\
? \text{ mL} = 2 \text{ tsp} \left( \frac{5 \text{ mL}}{1 \text{ tsp}} \right) = 10 \text{ mL}
\]

Given unit Answer with desired unit

Because 1 teaspoon is equivalent to 5 milliliters, multiplying by 5 mL/1 tsp is the same as multiplying by 1. The volume associated with 2 tsp does not change when we multiply by the conversion factor, but the value (number and unit) does. Because one milliliter is one-fifth the volume of one teaspoon, there are five times as many milliliters for a given volume. Therefore, 2 tsp and 10 mL represent the same volume.

Note that the units in a unit analysis setup cancel just like variables in an algebraic equation. Therefore, when we want to convert tsp to mL, we choose the ratio that has tsp on the bottom to cancel the tsp unit in our original value and leave us with the desired unit of mL. If you have used correct conversion factors, and if your units cancel to yield the desired unit or units, you can be confident that you will arrive at the correct answer.

**Metric-Metric Conversions**

As you saw in Chapter 1, one of the convenient features of the metric system is that the relationships between metric units can be derived from the metric prefixes. These relationships can easily be translated into conversion factors. For example, milli- means \(10^{-3}\) (or 0.001 or 1/1000), so a milliliter (mL) is \(10^{-3}\) liters (L). Thus there are 1000 or \(10^3\) milliliters in a liter. (A complete list of the prefixes that you need to know to solve the problems in this text is in Table 1.2.) Two possible sets of conversion factors for relating milliliters to liters can be obtained from these relationships.

\[
10^3 \text{ mL} = 1 \text{ L} \quad \text{leads to} \quad \frac{10^3 \text{ mL}}{1 \text{ L}} \quad \text{or} \quad \frac{1 \text{ L}}{10^3 \text{ mL}}
\]

\[
1 \text{ mL} = 10^{-3} \text{ L} \quad \text{leads to} \quad \frac{1 \text{ mL}}{10^{-3} \text{ L}} \quad \text{or} \quad \frac{10^{-3} \text{ L}}{1 \text{ mL}}
\]

In the remainder of this text, metric-metric conversion factors will have positive exponents like those found in the first set of conversion factors above.
**Example 2.1 - Conversion Factors**

**Objective 2**

Write two conversion factors that relate nanometers and meters. Use positive exponents in each.

**Solution**

Nano- means $10^{-9}$, so nanometer means $10^{-9}$ meters.

$$1 \text{ nm} = 10^{-9} \text{ m} \text{ and } 10^9 \text{ nm} = 1 \text{ m}$$

Because we want our conversion factors to have positive exponents, we will build our ratios from the equation on the right ($10^9 \text{ nm} = 1 \text{ m}$):

$$\frac{10^9 \text{ nm}}{1 \text{ m}} \text{ or } \frac{1 \text{ m}}{10^9 \text{ nm}}$$

**Exercise 2.1 - Conversion Factors**

**Objective 2**

Write two conversion factors that relate the following pairs of metric units. Use positive exponents for each.

- a. meter and kilometer
- b. meter and centimeter
- c. liter and gigaliter
- d. gram and microgram
- e. gram and megagram

**Example 2.2 - Unit Conversions**

**Objective 3**

Convert 365 nanometers to kilometers.

**Solution**

We want the answer in kilometers (km), and the units we are given are nanometers (nm), so, we are converting from one metric length unit to another metric length unit. We begin by writing

$$? \text{ km} = 365 \text{ nm}$$

We continue constructing the unit analysis setup by writing the “skeleton” of a conversion factor: the parentheses, the line dividing the numerator and the denominator, and the unit that we know we want to cancel. This step helps to organize our thoughts by showing us that our first conversion factor must have the nm unit on the bottom to cancel the nm unit associated with 365 nm.

Desired unit

$? \text{ km} = 365 \text{ nm}$

Skeleton

Given value

Unit to be cancelled
Note that in this problem both the desired metric unit and the known one have prefixes. One way to make this type of conversion is to change the given unit to the corresponding metric base unit, and then change that metric base unit to the desired unit. We write two conversion factors, one for each of these changes.

Sometimes it is useful to write a simple description of your plan first. Our plan in this instance is

\[ \text{nm} \rightarrow \text{m} \rightarrow \text{km} \]

The unit analysis setup would therefore be

\[
\frac{\text{Converst given metric unit to metric base unit}}{\text{Converts metric base unit to desired metric unit}} \quad ? \text{ km} = 365 \text{ nm} \left( \frac{1 \text{ m}}{10^9 \text{ nm}} \right) \left( \frac{1 \text{ km}}{10^3 \text{ m}} \right) = 3.65 \times 10^{-10} \text{ km}
\]

The \( \text{nm} \) and \( \text{m} \) units cancel, leaving the answer in \( \text{km} \).

**EXERCISE 2.2 - Unit Conversions**

Convert 4.352 micrograms to megagrams.

**English-Metric Conversions**

English units\(^2\) are still common in some countries, while people in other countries (and the scientific community everywhere), use metric units almost exclusively. Unit analysis provides a convenient method for converting between English and metric units. Several of the most commonly needed English-metric conversion factors are listed in Table 2.1 on the next page. Because the English inch is defined as 2.54 cm, the number 2.54 in this value is exact. The numbers in the other conversion factors in Table 2.1 are not exact.

\(^2\)Table A.2 in Appendix A shows some useful English-English conversion factors.
Sometimes the British are obstinate about the change from English to metric units. Greengrocer Steve Thoburn went to jail for refusing to switch from pounds to kilograms.

### Example 2.3 - Unit Conversions

The mass of a hydrogen atom is $1.67 \times 10^{-18}$ micrograms. Convert this mass into pounds.

**Solution**

We start with

$$\text{? lb } = 1.67 \times 10^{-18} \mu\text{g}$$

We then add the skeleton of the first conversion factor.

$$\text{? lb } = 1.67 \times 10^{-18} \mu\text{g} \left( \frac{1 \text{ g}}{10^6 \mu\text{g}} \right) \left( \frac{1 \text{ lb}}{453.6 \text{ g}} \right)$$

We are converting from metric mass to English mass. Table 2.1 contains a conversion factor that is convenient for most English-metric mass conversions, the one relating grams to pounds. Our given unit is micrograms. If we convert the micrograms into grams, we can then convert the gram unit into pounds.

$$\mu\text{g} \rightarrow \text{g} \rightarrow \text{lb}$$

Converts metric mass unit to English mass unit

$$\text{? lb } = 1.67 \times 10^{-18} \mu\text{g} \left( \frac{1 \text{ g}}{10^6 \mu\text{g}} \right) \left( \frac{1 \text{ lb}}{453.6 \text{ g}} \right) = 3.68 \times 10^{-27} \text{ lb}$$

### Exercise 2.3 - Conversion Factors

The volume of the Earth’s oceans is estimated to be $1.5 \times 10^{18}$ kiloliters. What is this volume in gallons?

### Table 2.1

**English-Metric Unit Conversion Factors**

<table>
<thead>
<tr>
<th>Type of measurement</th>
<th>Probably most useful to know</th>
<th>Also useful to know</th>
</tr>
</thead>
<tbody>
<tr>
<td>length</td>
<td>$\frac{2.54 \text{ cm}}{1 \text{ in.}}$ (exact)</td>
<td>$\frac{1.609 \text{ km}}{1 \text{ mi}}$</td>
</tr>
<tr>
<td>mass</td>
<td>$\frac{453.6 \text{ g}}{1 \text{ lb}}$</td>
<td>$\frac{2.205 \text{ lb}}{1 \text{ kg}}$</td>
</tr>
<tr>
<td>volume</td>
<td>$\frac{3.785 \text{ L}}{1 \text{ gal}}$</td>
<td>$\frac{1.057 \text{ qt}}{1 \text{ L}}$</td>
</tr>
</tbody>
</table>
Most of your calculations in chemistry are likely to be done using a calculator, and calculators often provide more digits in the answer than you would be justified in reporting as scientific data. This section shows you how to round off an answer to reflect the approximate range of certainty warranted by the data.

**Measurements, Calculations, and Uncertainty**

In Section 1.5, you read about the issue of uncertainty in measurement and learned to report measured values to reflect this uncertainty. For example, an inexpensive letter scale might show you that the mass of a nickel is 5 grams, but this is not an exact measurement. It is reasonable to assume that the letter scale measures mass with a precision of ±1 g and that the nickel therefore has a mass between 4 grams and 6 grams. You could use a more sophisticated instrument with a precision of ±0.01 g and report the mass of the nickel as 5.00 g. The purpose of the zeros in this value is to show that this measurement of the nickel’s mass has an uncertainty of plus or minus 0.01 g. With this instrument, we can assume that the mass of the nickel is between 4.99 g and 5.01 g. *Unless we are told otherwise, we assume that values from measurements have an uncertainty of plus or minus one in the last decimal place reported.* Using a far more precise balance found in a chemistry laboratory, you could determine the mass to be 4.9800 g, but this measurement still has an uncertainty of ±0.0001 g. *Measurements never give exact values.*

**Figure 2.1**

*Measurement Precision*

Even highly precise measurements have some uncertainty. Each of these balances yields a different precision for the mass of a nickel.

\[
\begin{align*}
\text{mass} & = 5.0 \text{ g} & \text{meaning} & 4.9 \text{ g to } 5.1 \text{ g} \\
\text{mass} & = 4.98 \text{ g} & \text{meaning} & 4.97 \text{ g to } 4.99 \text{ g} \\
\text{mass} & = 4.9800 \text{ g} & \text{meaning} & 4.9799 \text{ g to } 4.9801 \text{ g}
\end{align*}
\]
If a calculation is performed using all exact values and if the answer is not rounded off, the answer is exact, but this is a rare occurrence. The values used in calculations are usually *not* exact, and the answers should be expressed in a way that reflects the proper degree of uncertainty. Consider the conversion of the mass of our nickel from grams to pounds. (There are 453.6 g per pound.)

\[
? \text{ lb} = 4.9800 \text{ g} \left( \frac{1 \text{ lb}}{453.6 \text{ g}} \right) = 0.01098 \text{ lb (or } 1.098 \times 10^{-2} \text{ lb)}
\]

The number 4.9800 is somewhat uncertain because it comes from a measurement. The number 453.6 was derived from a calculation, and the answer to that calculation was rounded off to four digits. Therefore, the number 453.6 is also uncertain. Thus any answer we obtain using these numbers is inevitably going to be uncertain as well.

Different calculators or computers report different numbers of decimal places in their answers. For example, perhaps a computer reports the answer to 4.9800 divided by 453.6 as 0.01097883597884. If we were to report this result as the mass of our nickel, we would be suggesting that we were certain of the mass to a precision of ±0.00000000000001, which is not the case. Instead, we report 0.01098 lb (or 1.098 × 10^{-2} lb), which is a better reflection of the uncertainty in the numbers we used to calculate our answer.

### Rounding Off Answers Derived from Multiplication and Division

There are three general steps to rounding off answers so that they reflect the uncertainty of the values used in a calculation. Consider the example below, which shows how the mass of a hydrogen atom in micrograms can be converted into the equivalent mass in pounds.

\[
? \text{ lb} = 1.67 \times 10^{-18} \text{ μg} \left( \frac{1 \text{ g}}{10^6 \text{ μg}} \right)
\]

The first step in rounding off is to decide which of the numbers in the calculation affect the uncertainty of the answer. We can assume that 1.67 × 10^{-18} μg comes from a measurement, and all measurements are uncertain to some extent. Thus 1.67 × 10^{-18} affects the uncertainty of our answer. The 10^6 number comes from the definition of the metric prefix micro-, so it is exact. Because it has no effect on the uncertainty of our answer, we will not consider it when we are deciding how to round off our answer. The 453.6 comes from a calculation that was rounded off, so it is not exact. It affects the uncertainty of our answer and must be considered when we round our answer.

The second step in rounding off is to consider the degree of uncertainty in each of our inexact values. We can determine their relative uncertainties by counting the numbers of significant figures: three in 1.67 × 10^{-18} and four in 453.6. The number of **significant figures**, which is equal to the number of meaningful digits in a value, reflects the degree of uncertainty in the value (this is discussed more specifically in Study Sheet 2.1). A larger number of significant figures indicates a smaller uncertainty.

The final step is to round off our answer to reflect the most uncertain value used in our calculation. *When an answer is calculated by multiplying or dividing, we round it off to the same number of significant figures as the inexact value with the fewest significant figures.* For our example, that value is 1.67 × 10^{-18} μg, with three significant figures, so
we round off the calculated result, $3.681657848325 \times 10^{-27}$, to $3.68 \times 10^{-27}$.

The following sample study sheet provides a detailed guide to rounding off numbers calculated using multiplication and division. (Addition and subtraction will be covered in the subsequent discussion.) Examples 2.4 and 2.5 demonstrate these steps.

**TIP-OFF** After calculating a number using multiplication and division, you need to round it off to the correct number of significant figures.

**GENERAL STEPS**

**STEP 1** Determine whether each value is exact or not, and ignore exact values.
- Numbers that come from definitions are exact.
  - Numbers in metric-metric conversion factors that are derived from the metric prefixes are exact, such as
    \[
    \frac{10^3 \text{ g}}{1 \text{ kg}}
    \]
  - Numbers in English-English conversion factors with the same type of unit (for example, both length units) top and bottom are exact, such as
    \[
    \frac{12 \text{ in.}}{1 \text{ ft}}
    \]
  - The number 2.54 in the following conversion factor is exact.
    \[
    \frac{2.54 \text{ cm}}{1 \text{ in.}}
    \]
- Numbers derived from counting are exact. For example, there are exactly five toes in the normal foot.
  \[
  \frac{5 \text{ toes}}{1 \text{ foot}}
  \]
- Values that come from measurements are never exact.
- We will assume that values derived from calculations are not exact unless otherwise indicated. (With one exception, the numbers relating English to metric units that you will see in this text have been calculated and rounded, so they are not exact. The exception is 2.54 cm/1 in. The 2.54 comes from a definition.)

**STEP 2** Determine the number of significant figures in each value that is not exact.
- All non-zero digits are significant.

\[
\begin{align*}
1.35 & \quad \text{Five significant figures} \\
11.275 g & \quad \text{Five significant figures}
\end{align*}
\]
Objective 8

- **Zeros between nonzero digits are significant.**

  A zero between nonzero digits
  
  \[
  \begin{array}{c}
  \text{1} \\
  \text{3} \\
  \text{5} \\
  \text{10.275 g} \\
  \text{2} \\
  \text{4} \\
  \end{array}
  \quad \text{Five significant figures}
  \]

- **Zeros to the left of nonzero digits are not significant.**

  Not significant figures
  
  \[
  \begin{array}{c}
  \text{1} \\
  \text{3} \\
  \text{2} \\
  \text{0.000102 kg} \violet{\text{which can be described as}} \\
  \text{1} \\
  \text{3} \\
  \text{2} \\
  \text{1.02 \times 10^{-4} kg} \violet{\text{Both have three significant figures.}}
  \end{array}
  \]

- **Zeros to the right of nonzero digits in numbers that include decimal points are significant.**

  Five significant figures
  
  \[
  \begin{array}{c}
  \text{1} \\
  \text{3} \\
  \text{5} \\
  \text{1.000 g} \\
  \text{2} \\
  \text{4}
  \end{array}
  \quad \text{Three significant figures}
  \]

  Unnecessary for reporting size of value, but do reflect degree of uncertainty.

- **Zeros to the right of nonzero digits in numbers without decimal points are ambiguous for significant figures.**

  Precise to ±1 kg or ±10 kg?
  \[
  \begin{array}{c}
  \text{1} \\
  \text{2} \\
  \text{220 kg} \violet{\text{Two or three significant figures?}}
  \end{array}
  \]

  Important for reporting size of value, but unclear about degree of uncertainty
  
  \[
  \begin{array}{c}
  \text{1} \\
  \text{2} \\
  \text{2.2 \times 10^2 kg}
  \end{array}
  \]

  Use scientific notation to remove ambiguity.
  
  \[
  \begin{array}{c}
  \text{1} \\
  \text{2} \\
  \text{2.20 \times 10^2 kg}
  \end{array}
  \]

**Objective 8**

**Step 3** When multiplying and dividing, round your answer off to the same number of significant figures as the value containing the fewest significant figures.

- If the digit to the right of the final digit you want to retain is less than 5, round down (the last digit remains the same).

  \[
  \begin{array}{c}
  \text{26.221 rounded to three significant figures is 26.2} \\
  \text{First digit dropped is less than 5}
  \end{array}
  \]
If the digit to the right of the final digit you want to retain is 5 or greater, round up (the last significant digit increases by 1).

- 26.272 rounded to three significant figures is 26.3
  First digit dropped is greater than 5.

- 26.2529 rounded to three significant figures is 26.3
  First digit dropped is equal to 5.

- 26.15 rounded to three significant figures is 26.2
  First digit dropped is equal to 5.

**Example**  See Examples 2.4 and 2.5.

**Example 2.4 - Rounding Off Answers Derived from Multiplication and Division**

The average human body contains 5.2 L of blood. What is this volume in quarts? The unit analysis setup for this conversion is below. Identify whether each value in the setup is exact or not. Determine the number of significant figures in each inexact value, calculate the answer, and report it to the correct number of significant figures.

\[
? \text{ qt} = 5.2 \text{ L} \left( \frac{1 \text{ gal}}{3.785 \text{ L}} \right) \left( \frac{4 \text{ qt}}{1 \text{ gal}} \right)
\]

**Solution**

A typical calculator shows the answer to this calculation to be 5.4953766, a number with far too many decimal places, considering the uncertainty of the values used in the calculation. It needs to be rounded to the correct significant figures.

**Step 1** The 5.2 L is based on measurement, so it is not exact. The 3.785 L is part of an English-metric conversion factor, and we assume those factors are not exact except for 2.54 cm/in. On the other hand, 4 qt/gal is an English-English conversion factor based on the definition of quart and gallon; thus the 4 is exact.

**Step 2** Because 5.2 contains two nonzero digits, it has two significant figures. The number 3.785 contains four nonzero digits, so it has four significant figures.

**Step 3** Because the value with the fewest significant figures has two significant figures, we report two significant figures in our answer, rounding 5.4953766 to 5.5.

\[
? \text{ qt} = 5.2 \text{ L} \left( \frac{1 \text{ gal}}{3.785 \text{ L}} \right) \left( \frac{4 \text{ qt}}{1 \text{ gal}} \right) = 5.5 \text{ qt}
\]
How many minutes does it take an ant walking at 0.01 m/s to travel 6.0 feet across a picnic table? The unit analysis setup for this conversion is below. Identify whether each value in the setup is exact or not. Determine the number of significant figures in each inexact value, calculate the answer, and report it to the correct number of significant figures.

\[
? \text{ min} = 6.0 \text{ ft} \left( \frac{12 \text{ in.}}{1 \text{ ft}} \right) \left( \frac{2.54 \text{ cm}}{1 \text{ in.}} \right) \left( \frac{1 \text{ m}}{10^2 \text{ cm}} \right) \left( \frac{1 \text{ s}}{0.01 \text{ m}} \right) \left( \frac{1 \text{ min}}{60 \text{ s}} \right)
\]

**Solution**

**Step 1** The table's length and the ant’s velocity come from measurements, so 6.0 and 0.01 are not exact. The other numbers are exact because they are derived from definitions. Thus, only 6.0 and 0.01 can limit our significant figures.

**Step 2** Zeros to the right of nonzero digits in numbers that have decimal points are significant, so 6.0 contains two significant figures. Zeros to the left of nonzero digits are not significant, so 0.01 contains one significant figure.

**Step 3** A typical calculator shows 3.048 for the answer. Because the value with the fewest significant figures has one significant figure, we report one significant figure in our answer. Our final answer of 3 minutes signifies that it could take 2 to 4 minutes for the ant to cross the table.

\[
? \text{ min} = 6.0 \text{ ft} \left( \frac{12 \text{ in.}}{1 \text{ ft}} \right) \left( \frac{2.54 \text{ cm}}{1 \text{ in.}} \right) \left( \frac{1 \text{ m}}{10^2 \text{ cm}} \right) \left( \frac{1 \text{ s}}{0.01 \text{ m}} \right) \left( \frac{1 \text{ min}}{60 \text{ s}} \right) = 3 \text{ min}
\]
The re-entry speed of the Apollo 10 space capsule was 11.0 km/s. How many hours would it have taken for the capsule to fall through 25.0 miles of the stratosphere? The unit analysis setup for this calculation is below. Identify whether each value in the setup is exact or not. Determine the number of significant figures in each inexact value, calculate the answer, and report it to the correct number of significant figures.

\[ ? \text{ hr} = 25.0 \text{ mi} \left( \frac{5280 \text{ ft}}{1 \text{ mi}} \right) \left( \frac{12 \text{ in}}{1 \text{ ft}} \right) \left( \frac{2.54 \text{ cm}}{1 \text{ in}} \right) \left( \frac{1 \text{ m}}{10^2 \text{ cm}} \right) \left( \frac{1 \text{ km}}{10^3 \text{ m}} \right) \left( \frac{1 \text{ s}}{11.0 \text{ km}} \right) \left( \frac{1 \text{ min}}{60 \text{ s}} \right) \left( \frac{1 \text{ hr}}{60 \text{ min}} \right) \]

**Rounding Off Answers Derived from Addition and Subtraction**

The following sample study sheet provides a guide to rounding off numbers calculated using addition and subtraction.

**Tip-off** After calculating a number using addition and subtraction, you need to round it off to the correct number of decimal positions.

**General Steps**

**Step 1** Determine whether each value is exact, and ignore exact values (see Study Sheet 2.1).

**Step 2** Determine the number of decimal places for each value that is not exact.

**Step 3** Round your answer to the same number of decimal places as the inexact value with the fewest decimal places.

**Example** See Example 2.6.
A laboratory procedure calls for you to determine the mass of an unknown liquid. Let’s suppose that you weigh a 100-mL beaker on a new electronic balance and record its mass as 52.3812 g. You then add 10 mL of the unknown liquid to the beaker and discover that the electronic balance has stopped working. You find a 30-year-old balance in a cupboard, use it to weigh the beaker of liquid, and record that mass as 60.2 g. What is the mass of the unknown liquid?

**Solution**

You can calculate the mass of the liquid by subtracting the mass of the beaker from the mass of the beaker and the liquid.

\[
60.2 \text{ g beaker with liquid} - 52.3812 \text{ g beaker} = 7.8188 \text{ g liquid}
\]

We can use the steps outlined in Sample Study Sheet 2.2 to decide how to round off our answer.

**Step 1** The numbers 60.2 and 52.3812 come from measurements, so they are not exact.

**Step 2** We assume that values given to us have uncertainties of ±1 in the last decimal place reported, so 60.2 has an uncertainty of ±0.1 g and 52.3812 has an uncertainty of ±0.0001 g. The first value is precise to the tenths place, and the second value is precise to four places to the right of the decimal point.

**Step 3** We round answers derived from addition and subtraction to the same number of decimal places as the value with the fewest. Therefore, we report our answer to the tenth’s place—rounding it off if necessary—to reflect this uncertainty. The answer is **7.8 g**.

Be sure to remember that the guidelines for rounding answers derived from addition or subtraction are different from the guidelines for rounding answers from multiplication or division. Notice that when we are adding or subtracting, we are concerned with decimal places in the numbers used rather than with the number of significant figures. Let’s take a closer look at why. In Example 2.6, we subtracted the mass of a beaker (52.3812 g) from the mass of the beaker and an unknown liquid (60.2 g) to get the mass of the liquid. If the reading of 60.2 has an uncertainty of ±0.1 g, the actual value could be anywhere between 60.1 g and 60.3 g. The range of possible values for the mass of the beaker is 52.3811 g to 52.3813 g. This leads to a range of possible values for our answer from 7.7187 g to 7.9189 g.

\[
\begin{array}{c|c|c|c}
60.1 & 60.3 \\
-52.3813 & -52.3811 \\
7.7187 & 7.9189 \\
\end{array}
\]

Note that our possible values vary from about 7.7 g to about 7.9 g, or ±0.1 of our reported answer of 7.8 g. Because our least precise value (60.2 g) has an uncertainty of ±0.1 g, our answer can be no more precise than ±0.1 g.
Use the same reasoning to prove that the following addition and subtraction problems are rounded to the correct number of decimal positions.

\[ 97.40 + 31 = 128 \]
\[ 1035.67 - 989.2 = 46.5 \]

Note that although the numbers in the addition problem have four and two significant figures, the answer is reported with three significant figures. This answer is limited to the ones place by the number 31, which we assume has an uncertainty of ±1. Note also that although the numbers in the subtraction problem have six and four significant figures, the answer has only three. The answer is limited to the tenths place by 989.2, which we assume has an uncertainty of ±0.1.

**Objective 9**

**Exercise 2.6 - Rounding Off Answers Derived from Addition and Subtraction**

Report the answers to the following calculations to the correct number of decimal positions. Assume that each number is ±1 in the last decimal position reported.

a. \[ 684 - 595.325 = \]

b. \[ 92.771 + 9.3 = \]

When people say that lead is heavier than wood, they do not mean that a pea-sized piece of lead weighs more than a truckload of pine logs. What they mean is that a sample of lead will have a greater mass than an equal volume of wood. A more concise way of putting this is that lead is more dense than wood. This type of density, formally known as mass density, is defined as mass divided by volume. It is what people usually mean by the term **density**.

\[
\text{Density} = \frac{\text{mass}}{\text{volume}}
\]

The density of lead is 11.34 g/mL, and the density of pinewood is about 0.5 g/mL. In other words, a milliliter of lead contains 11.34 g of matter, while a milliliter of pine contains only about half a gram of matter. See Table 2.2 on the next page for the densities of other common substances.

Although there are exceptions, the densities of liquids and solids generally decrease with increasing temperature\(^3\). Thus, when chemists report densities, they generally state the temperature at which the density was measured. For example, the density of ethanol is 0.806 g/mL at 0 °C but 0.789 g/mL at 20 °C.\(^4\)

\(^3\)The density of liquid water actually increases as its temperature rises from 0 °C to 4 °C. Such exceptions are very rare.

\(^4\)The temperature effect on the density of gases is more complicated, but it, too, changes with changes in temperature. This effect will be described in Chapter 11.
The densities of liquids and solids are usually described in grams per milliliter or grams per cubic centimeter. (Remember a milliliter and a cubic centimeter are the same volume.) The particles of a gas are much farther apart than the particles of a liquid or solid, so gases are much less dense than solids and liquids. Thus it is more convenient to describe the densities of gases as grams per liter. The density of air at sea level and 20 °C is about 1.2 g/L.

Because the density of a substance depends on the substance’s identity and its temperature, it is possible to identify an unknown substance by comparing its density at a particular temperature to the densities of known substances at the same temperature. For example, we can determine whether an object is pure gold by measuring its density at 20 °C and comparing that to the known density of gold at 20 °C, 19.31 g/mL. Figure 2.2 shows densities of some common substances.

Table 2.2
Mass Densities of Some Common Substances (at 20 °C unless otherwise stated)

<table>
<thead>
<tr>
<th>Substance</th>
<th>Density, g/mL</th>
</tr>
</thead>
<tbody>
<tr>
<td>air at sea level</td>
<td>0.0012 (or 1.2 g/L)</td>
</tr>
<tr>
<td>Styrofoam</td>
<td>0.03</td>
</tr>
<tr>
<td>pinewood</td>
<td>0.4-0.6</td>
</tr>
<tr>
<td>gasoline</td>
<td>0.70</td>
</tr>
<tr>
<td>ethanol</td>
<td>0.7893</td>
</tr>
<tr>
<td>ice</td>
<td>0.92</td>
</tr>
<tr>
<td>water, H₂O, at 20 °C</td>
<td>0.9982</td>
</tr>
<tr>
<td>water, H₂O, at 0 °C</td>
<td>0.9998</td>
</tr>
<tr>
<td>water, H₂O, at 3.98 °C</td>
<td>1.0000</td>
</tr>
<tr>
<td>seawater</td>
<td>1.025</td>
</tr>
<tr>
<td>whole blood</td>
<td>1.05</td>
</tr>
<tr>
<td>bone</td>
<td>1.5-2.0</td>
</tr>
<tr>
<td>glass</td>
<td>2.4-2.8</td>
</tr>
<tr>
<td>aluminum, Al</td>
<td>2.702</td>
</tr>
<tr>
<td>the planet Earth (average)</td>
<td>5.25</td>
</tr>
<tr>
<td>iron, Fe</td>
<td>7.86</td>
</tr>
<tr>
<td>silver, Ag</td>
<td>10.5</td>
</tr>
<tr>
<td>gold, Au</td>
<td>19.31</td>
</tr>
<tr>
<td>lead, Pb</td>
<td>11.34</td>
</tr>
<tr>
<td>platinum, Pt</td>
<td>21.45</td>
</tr>
<tr>
<td>atomic nucleus</td>
<td>≈10^{14}</td>
</tr>
<tr>
<td>a black hole (not 20 °C)</td>
<td>≈10^{16}</td>
</tr>
</tbody>
</table>
Using Density as a Conversion Factor

Because density is reported as a ratio that describes a relationship between two units, the density of a substance can be used in unit analysis to convert between the substance's mass and its volume. Examples 2.7 and 2.8 show how the density of water at 20 °C can be used to convert between the mass in grams of a given sample of water and the sample's volume in milliliters.

**Example 2.7 - Density Conversions**

What is the mass in grams of 75.0 mL of water at 20 °C?

*Solution*

The unit analysis setup for this problem begins

\[ \text{75.0 mL H}_2\text{O} \times \frac{? \text{ g H}_2\text{O}}{1 \text{ mL H}_2\text{O}} = 75.0 \text{ mL H}_2\text{O} \times \frac{0.9982 \text{ g H}_2\text{O}}{1 \text{ mL H}_2\text{O}} \]

Being asked to convert from volume into mass is the tip-off that we can use the density of water as a conversion factor in solving this problem. We can find water's density on a table of densities, such as Table 2.2.

\[ \text{The density of water at 20 °C} \]

\[ \text{75.0 mL H}_2\text{O} \times \frac{0.9982 \text{ g H}_2\text{O}}{1 \text{ mL H}_2\text{O}} = 74.9 \text{ g H}_2\text{O} \]

**Example 2.8 - Density Conversions**

What is the volume of 25.00 kilograms of water at 20 °C?

*Solution*

Like any other conversion factor, density can be used in the inverted form to make conversions:

\[ \frac{0.9982 \text{ g H}_2\text{O}}{1 \text{ mL}} \quad \text{or} \quad \frac{1 \text{ mL}}{0.9982 \text{ g H}_2\text{O}} \]

The conversion factor on the right allows us to convert from mass in grams to volume in milliliters. First, however, we need to convert the given unit, kilograms, to grams. Then, after using the density of water to convert grams to milliliters, we need to convert milliliters to liters.

\[ \text{25.00 kg H}_2\text{O} \times \frac{10^3 \text{ g}}{1 \text{ kg}} \times \frac{1 \text{ mL H}_2\text{O}}{0.9982 \text{ g H}_2\text{O}} \times \frac{1 \text{ L}}{10^3 \text{ mL}} = 25.05 \text{ L H}_2\text{O} \]
**Exercise 2.7 - Density Conversions**

**Objective 11**

- What is the mass in kilograms of 15.6 gallons of gasoline?
- A shipment of iron to a steel-making plant has a mass of 242.6 metric tons. What is the volume in liters of this iron?

**Determination of Mass Density**

The density of a substance can be calculated from its measured mass and volume. The examples that follow show how this can be done and demonstrate more of the unit analysis thought process.

**Example 2.9 - Density Calculations**

An empty 2-L graduated cylinder is found to have a mass of 1124.2 g. Liquid methanol, CH₃OH, is added to the cylinder, and its volume measured as 1.20 L. The total mass of the methanol and the cylinder is 2073.9 g, and the temperature of the methanol is 20 °C. What is the density of methanol at this temperature?

**Solution**

We are not told the specific units desired for our answer, but we will follow the usual convention of describing the density of liquids in grams per milliliter. It is a good idea to write these units in a way that reminds us we want $g$ on the top of our ratio when we are done and $mL$ on the bottom.

\[
\frac{? \text{ g}}{\text{mL}} = \]

Because we want our answer to contain a ratio of two units, we start the right side of our setup with a ratio of two units. Because we want mass on the top when we are done and volume on the bottom, we put our mass unit on the top and our volume unit on the bottom. The mass of the methanol is found by subtracting the mass of the cylinder from the total mass of the cylinder and the methanol.

\[
\frac{? \text{ g}}{\text{mL}} = \frac{(2073.9 - 1124.2) \text{ g}}{1.20 \text{ L}}
\]

Now that we have placed units for the desired properties (mass and volume) in the correct positions, we convert the units we have been given to the specific units we want. For our problem, this means converting liters to milliliters. Because we want to cancel $L$ and it is on the bottom of the first ratio, the skeleton of the next conversion factor has the $L$ on top.

\[
\frac{? \text{ g}}{\text{mL}} = \frac{(2073.9 - 1124.2) \text{ g}}{1.20 \text{ L}} \left( \frac{\text{L}}{1000 \text{ mL}} \right)
\]

The completed setup, and the answer, are

\[
\frac{? \text{ g}}{\text{mL}} = \frac{(2073.9 - 1124.2) \text{ g}}{1.20 \text{ L}} \left( \frac{1 \text{ L}}{1000 \text{ mL}} \right) = \frac{949.7 \text{ g}}{1.20 \text{ L}} \left( \frac{1 \text{ L}}{1000 \text{ mL}} \right) = 0.791 \text{ g/mL}
\]
**Example 2.10 - Density Calculations**

You could find out whether a bracelet is made of silver or platinum by determining its density and comparing it to the densities of silver (10.5 g/mL) and platinum (21.45 g/mL). A bracelet is placed directly on the pan of a balance, and its mass is found to be 50.901 g. Water is added to a graduated cylinder, and the volume of the water is recorded as 18.2 mL. The bracelet is added to the water, and the volume rises to 23.0 mL. What is the density of the bracelet? Is it more likely to be silver or platinum?

**Solution**

We can find the volume of the bracelet by subtracting the volume of the water from the total volume of water and bracelet. We can then calculate the density using the following setup.

\[
\frac{? \text{ g bracelet}}{\text{mL bracelet}} = \frac{50.901 \text{ g bracelet}}{(23.0 - 18.2) \text{ mL bracelet}}
\]

Rounding off an answer that is derived from a mixture of subtraction (or addition) and division (or multiplication) is more complex than when these calculations are done separately. We need to recognize the different components of the problem and follow the proper rules for each.

Because 23.0 and 18.2 are both uncertain in the tenth's place, the answer from the subtraction is reported to the tenth's place only. This answer is 4.8.

When we divide 50.901, which has five significant figures, by 4.8, which has two significant figures, we report two significant figures in our answer.

\[
\frac{? \text{ g bracelet}}{\text{mL bracelet}} = \frac{50.901 \text{ g bracelet}}{(23.0 - 18.2) \text{ mL bracelet}} = \frac{50.901 \text{ g bracelet}}{4.8 \text{ mL bracelet}} = 11 \text{ g/mL bracelet}
\]

The density is closer to that of silver, so the bracelet is more likely to be silver than platinum.

**Exercise 2.8 - Density Calculations**

a. A graduated cylinder is weighed and found to have a mass of 48.737 g. A sample of hexane, C\textsubscript{6}H\textsubscript{14}, is added to the graduated cylinder, and the total mass is measured as 57.452 g. The volume of the hexane is 13.2 mL. What is the density of hexane?

b. A tree trunk is found to have a mass of $1.2 \times 10^4$ kg and a volume of $2.4 \times 10^4$ L. What is the density of the tree trunk in g/mL?
2.4 Percentage and Percentage Calculations

What does it mean to say that your body is about 8.0% blood, and that when you work hard, between 3% and 4% by volume of your blood goes to your brain? Once you understand their meaning, percentage values such as these will provide you with ratios that can be used as conversion factors.

*Percentage by mass,* the most common form of percentage used in chemical descriptions, is a value that tells us the number of mass units of the part for each 100 mass units of the whole. You may assume that any percentage in this book is a percentage by mass unless you are specifically told otherwise. Thus, when we are told that our bodies are 8.0% blood, we assume that means 8.0% by mass, which is to say that for every 100 grams of body, there are 8.0 grams of blood and that for every 100 kilograms of body, there are 8.0 kilograms of blood. We can use any mass units we want in the ratio as long as the units are the same for the part and for the whole. Consequently, a percentage by mass can be translated into any number of conversion factors.

\[
\frac{8.0 \text{ kg blood}}{100 \text{ kg body}} \quad \text{or} \quad \frac{8.0 \text{ g blood}}{100 \text{ g body}} \quad \text{or} \quad \frac{8.0 \text{ lb blood}}{100 \text{ lb body}} \quad \text{or} \quad \ldots
\]

The general form for conversion factors derived from mass percentages is

**Objective 13**

For X% by mass

\[
\frac{X \text{ (any mass unit) part}}{100 \text{ (same mass unit) whole}}
\]

Another frequently encountered form of percentage is *percentage by volume* (or % by volume). Because we assume that all percentages in chemistry are mass percentages unless told otherwise, volume percentages should always be designated as such. The general form for conversion factors derived from volume percentages is

**Objective 13**

For X% by volume

\[
\frac{X \text{ (any volume unit) part}}{100 \text{ (same volume unit) whole}}
\]

For example, the statement that 3.2% by volume of your blood goes to your brain provides you with conversion factors to convert between volume of blood to the brain and volume of blood total.

\[
\frac{3.2 \text{ L blood to brain}}{100 \text{ L blood total}} \quad \text{or} \quad \frac{3.2 \text{ mL blood to brain}}{100 \text{ mL blood total}} \quad \text{or} \quad \frac{3.2 \text{ qt blood to brain}}{100 \text{ qt blood total}} \quad \text{or} \quad \ldots
\]
Your body is about 8.0% blood. If you weigh 145 pounds, what is the mass of your blood in kilograms?

**Solution**

Our setup begins with

\[ \text{? kg blood} = 145 \text{ lb body} \left( \frac{\text{lb body}}{\text{lb body}} \right) \]

Because we are not told the type of percentage that “8.0% blood” represents, we assume that it is a mass percentage. Both pounds and kilograms are mentioned in the problem, so we could use either one of the following conversion factors.

\[ \frac{8.0 \text{ lb blood}}{100 \text{ lb body}} \quad \text{or} \quad \frac{8.0 \text{ kg blood}}{100 \text{ kg body}} \]

Both of the following setups lead to the correct answer.

\[ \text{? kg blood} = 145 \text{ lb body} \left( \frac{8.0 \text{ lb blood}}{100 \text{ lb body}} \right) \left( \frac{453.6 \text{ g}}{1 \text{ lb}} \right) \left( \frac{1 \text{ kg}}{10^3 \text{ g}} \right) = 5.3 \text{ kg blood} \]

or

\[ \text{? kg blood} = 145 \text{ lb body} \left( \frac{453.6 \text{ g}}{1 \text{ lb}} \right) \left( \frac{1 \text{ kg}}{10^3 \text{ g}} \right) \left( \frac{8.0 \text{ kg blood}}{100 \text{ kg body}} \right) = 5.3 \text{ kg blood} \]

8.0% limits our significant figures to two.

**Exercise 2.9 - Unit Conversions**

a. The mass of the ocean is about \(1.8 \times 10^{21}\) kg. If the ocean contains 0.014% by mass hydrogen carbonate ions, \(\text{HCO}_3^-\), how many pounds of \(\text{HCO}_3^-\) are in the ocean?

b. When you are doing heavy work, your muscles get about 75 to 80% by volume of your blood. If your body contains 5.2 liters of blood, how many liters of blood are in your muscles when you are working hard enough to send them 78% by volume of your blood?
2.5 A Summary of the Unit Analysis Process

The winds and waves are always on the side of the ablest navigators.

Edward Gibson, English Historian

You have seen some of the many uses of unit analysis and looked at various kinds of information that provide useful conversion factors for chemical calculations. Now, it is time for you to practice a general procedure for navigating your way through unit conversion problems so that you will be able to do them efficiently on your own. Sample Study Sheet 2.3 describes a stepwise thought process that can help you to decide what conversion factors to use and how to assemble them into a unit analysis format.

Sample Study Sheet 2.3

Calculations Using Unit Analysis

**Objective 15**

**Tip-off** You wish to express a given value in terms of a different unit or units.

**General Steps**

**Step 1** State your question in an expression that sets the unknown unit(s) equal to one or more of the values given.

- To the left of the equals sign, show the unit(s) you want in your answer.
- To the right of the equals sign, start with an expression composed of the given unit(s) that parallels in kind and placement the units you want in your answer.

If you want a single unit in your answer, start with a value that has a single unit.

If you want a ratio of two units in your answer, start with a value that has a ratio of two units, or start with a ratio of two values, each of which has one unit. Put each type of unit in the position you want it to have in the answer.

**Step 2** Multiply the expression to the right of the equals sign by conversion factors that cancel unwanted units and generate the desired units.

If you are not certain which conversion factor to use, ask yourself, “What is the fundamental conversion the problem requires, and what conversion factor do I need to make that type of conversion?” Figure 2.3 provides a guide to useful conversion factors.

**Step 3** Do a quick check to be sure you used correct conversion factors and that your units cancel to yield the desired unit(s).

**Step 4** Do the calculation, rounding your answer to the correct number of significant figures and combining it with the correct unit.

**Example** See Examples 2.12 to 2.16.
### Figure 2.3
**Types of Unit Conversions**

Here is a summary of some of the basic types of conversions that are common in chemistry and the types of conversion factors used to make them.

<table>
<thead>
<tr>
<th>Metric unit</th>
<th>Another metric unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>(same type of measurement)</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>English length unit</th>
<th>Metric length unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>or conversion factors in Table 2.1</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>English mass unit</th>
<th>Metric mass unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>or conversion factors in Table 2.1</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>English volume unit</th>
<th>Metric volume unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>or conversion factors in Table 2.1</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>English unit</th>
<th>Another English unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>(same type of measurement)</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Mass</th>
<th>Volume</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density as a conversion factor</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Mass of a part</th>
<th>Mass of a whole</th>
</tr>
</thead>
<tbody>
<tr>
<td>For X% by mass</td>
<td>X (any mass unit) part</td>
</tr>
<tr>
<td>100 (same mass unit) whole</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Volume of a part</th>
<th>Volume of a whole</th>
</tr>
</thead>
<tbody>
<tr>
<td>For X% by volume</td>
<td>X (any volume unit) part</td>
</tr>
<tr>
<td>100 (same volume unit) whole</td>
<td></td>
</tr>
</tbody>
</table>
Here are more examples of the most useful types of unit analysis conversions.

**Example 2.12 - Metric-Metric Unit Conversions**

Convert 4567.36 micrograms to kilograms.

**Solution**

When converting from one metric unit to another, convert from the given unit to the base unit and then from the base unit to the unit that you want.

\[
\text{Desired unit} \quad \frac{1 \text{ g}}{10^6 \text{ g}} \left( \frac{1 \text{ kg}}{10^3 \text{ g}} \right) = 4.56736 \times 10^{-6} \text{ kg}
\]

**Example 2.13 - English-Metric Unit Conversions**

Convert 475 miles to kilometers.

**Solution**

The conversion factor 2.54 cm/in. can be used to convert from an English to a metric unit of length.

\[
\text{km} = 475 \text{ mi} \left( \frac{5280 \text{ ft}}{1 \text{ mi}} \right) \left( \frac{12 \text{ in.}}{1 \text{ ft}} \right) \left( \frac{2.54 \text{ cm}}{1 \text{ in.}} \right) \left( \frac{1 \text{ m}}{10^2 \text{ cm}} \right) \left( \frac{1 \text{ km}}{10^3 \text{ m}} \right) = 764 \text{ km}
\]

Memorizing other English-metric conversion factors will save you time and effort. For example, if you know that 1.609 km = 1 mi, the problem becomes much easier.

\[
\text{km} = 475 \text{ mi} \left( \frac{1.609 \text{ km}}{1 \text{ mi}} \right) = 764 \text{ km}
\]

**Example 2.14 - Unit Conversions Using Density**

What is the volume in liters of 64.567 pounds of ethanol at 20 °C?

**Solution**

Pound is a mass unit, and we want volume. Density provides a conversion factor that converts between mass and volume. You can find the density of ethanol on a table such as Table 2.2. It is 0.7893 g/mL at 20 °C.

\[
? \text{ L} = 64.567 \text{ lb} \left( \frac{453.6 \text{ g}}{1 \text{ lb}} \right) \left( \frac{1 \text{ mL}}{0.7893 \text{ g}} \right) \left( \frac{1 \text{ L}}{10^3 \text{ mL}} \right) = 37.11 \text{ L}
\]
Before we go on to the next example, let's look at one more way to generate unit analysis conversion factors. Anything that can be read as “something per something” can be used as a unit analysis conversion factor. For example, if a car is moving at 55 miles per hour, we could use the following conversion factor to convert from distance traveled in miles to time in hours or time in hours to distance traveled in miles.

\[
\left( \frac{55 \text{ mi}}{1 \text{ hr}} \right)
\]

If you are building a fence, and plan to use four nails per board, the following conversion factor allows you to calculate the number of nails necessary to nail up 94 fence boards.

\[
\left( \frac{4 \text{ nails}}{1 \text{ fence board}} \right)
\]

**Example 2.15 - Unit Conversions Using Percentage**

The label on a can of cat food tells you there are 0.94 lb of cat food per can with 0.15% calcium. If there are three servings per can, how many grams of calcium are in each serving?

**Solution**

Note that two phrases in this question can be read as “something per something” and therefore can be used as a unit analysis conversion factors. The phrase “three servings per can” leads to the first conversion factor used below, and “0.94 lb of cat food per can” leads to the second.

Percentages also provide ratios that can be used as unit analysis conversion factors. Because percentages are assumed to be mass percentages unless otherwise indicated, they tell us the number of mass units of the part for each 100 mass units of the whole. The ratio can be constructed using any unit of mass as long as the same unit is written in both the numerator and denominator. This leads to the third conversion factor in our setup. The fourth conversion factor changes pounds to grams.

\[
\text{? g Ca} = \text{One serv.} \left( \frac{1 \text{ can}}{3 \text{ serv.}} \right) \left( \frac{0.94 \text{ lb food}}{1 \text{ can}} \right) \left( \frac{0.15 \text{ lb Ca}}{100 \text{ lb food}} \right) \left( \frac{453.6 \text{ g}}{1 \text{ lb}} \right) = 0.21 \text{ g Ca}
\]

**Example 2.16 - Converting a Ratio of Two Units**

When 2.3942 kg of the sugar glucose are burned (combusted), 37,230 kJ of heat are evolved. What is the heat of combustion of glucose in J/g? (Heat evolved is described with a negative sign.)

**Solution**

When the answer you want is a ratio of two units, start your unit analysis setup with a ratio of two units. Put the correct type of unit in the correct position in the ratio. For this problem, we put the heat unit on the top and the mass unit on the bottom.

\[
\frac{? \text{ J}}{\text{g glucose}} = \frac{-37,230 \text{ kJ}}{2.3942 \text{ kg glucose}} \left( \frac{10^3 \text{ J}}{1 \text{ kJ}} \right) \left( \frac{1 \text{ kg}}{10^3 \text{ g}} \right) = -15,550 \text{ J/g}
\]
Exercise 2.10 - Unit Conversions

a. The diameter of a proton is $2 \times 10^{-15}$ meters. What is this diameter in nanometers?

b. The mass of an electron is $9.1093897 \times 10^{-31}$ kg. What is this mass in nanograms?

c. There are $4.070 \times 10^6$ lb of sulfuric acid used to make Jell-O each year. Convert this to kilograms.

d. A piece of Styrofoam has a mass of 88.978 g and a volume of 2.9659 L. What is its density in g/mL?

e. The density of blood plasma is 1.03 g/mL. A typical adult has about 2.5 L of blood plasma. What is the mass in kilograms of this amount of blood plasma?

f. Pain signals are transferred through the nervous system at a speed between 12 and 30 meters per second. If a student drops a textbook on her toe, how long will it take for the signal, traveling at a velocity of 18 meters per second, to reach her brain 6.0 feet away?

g. An electron takes $6.2 \times 10^{-9}$ seconds to travel across a TV set that is 22 inches wide. What is the velocity of the electron in km/hr?

h. The mass of the ocean is about $1.8 \times 10^{21}$ kg. If the ocean contains 0.041% by mass calcium ions, Ca$^{2+}$, how many tons of Ca$^{2+}$ are in the ocean? (There are 2000 pounds per ton.)

i. When you are at rest, your heart pumps about 5.0 liters of blood per minute. Your brain gets about 15% by volume of your blood. What volume of blood in liters is pumped through your brain in 1.0 hour of rest?

2.6 Temperature Conversions

Section 1.4 presented the three most frequently used scales for describing temperature: Celsius, Fahrenheit, and Kelvin. In this section, we will use the following equations to convert a temperature reported in one of these systems to the equivalent temperature in another. Note that the numbers 1.8, 32, and 273.15 in these equations all come from definitions, so they are all exact.

\[ ? ^\circ F = \text{number of } ^\circ C \left( \frac{1.8 ^\circ F}{1 ^\circ C} \right) + 32 ^\circ F \]

\[ ? ^\circ C = (\text{number of } ^\circ F - 32 ^\circ F) \left( \frac{1 ^\circ C}{1.8 ^\circ F} \right) \]

\[ ? K = \text{number of } ^\circ C + 273.15 \]

\[ ? ^\circ C = \text{number of } K - 273.15 \]
**Example 2.17 - Temperature Conversions**

“Heavy” water contains the heavy form of hydrogen called deuterium, whose atoms each have one proton, one neutron, and one electron. Heavy water freezes at 38.9 °F. What is this temperature in °C?

**Solution**

We use the equation for converting Fahrenheit temperatures to Celsius:

\[ \text{°C} = (\text{°F} - 32) \left( \frac{1}{1.8} \right) \]

Rounding off the answer can be tricky here. When you subtract 32 from 38.9, you get 6.9. The 32 is exact, so it is ignored when considering how to round off the answer. The 38.9 is precise to the first number after the decimal point, so the answer to the subtraction is reported to the tenths place. There are two significant figures in 6.9, so when we divide by the exact value of 1.8 °F, we round our answer to two significant figures.

\[ \text{°C} = (38.9 - 32) \left( \frac{1}{1.8} \right) = 6.9 \left( \frac{1}{1.8} \right) = 3.8 °C \]

**Example 2.18 - Temperature Conversions**

The compound 1-chloropropane, CH₃CH₂CH₂Cl, melts at 46.6 °C. What is this temperature in °F?

**Solution**

The equation for converting Celsius temperatures to Fahrenheit is

\[ \text{°F} = 46.6 \left( \frac{1.8}{1} \right) + 32 °F \]

Because the calculation involves multiplication and division as well as addition, you need to apply two different rules for rounding off your answer. When you multiply 46.6, which has three significant figures, by the exact value of 1.8 °F, your answer should have three significant figures. The answer on the display of the calculator, 83.88, would therefore be rounded off to 83.9. You then add the exact value of 32 °F and round off that answer to the tenths place.

\[ \text{°F} = 46.6 \left( \frac{1.8}{1} \right) + 32 °F = 83.9 °F + 32 °F = 115.9 °F \]
EXAMPLE 2.19 - Temperature Conversions

Silver melts at 961 °C. What is this temperature in K?

Solution

\[ ? \text{ K} = 961 \text{ °C} + 273.15 = 1234 \text{ K} \]

For rounding off our answer, we assumed that 961 °C came from a measurement and so is not exact. It is precise to the ones place. On the other hand, 273.15 is exact, and has no effect on the uncertainty of our answer. We therefore report the answer for our addition to the ones place, rounding off 1234.15 to 1234.

EXAMPLE 2.20 - Temperature Conversions

Tin(II) sulfide, SnS, melts at 1155 K. What is this temperature in °C?

Solution

\[ ? \text{ °C} = 1155 \text{ K} - 273.15 = 882 \text{ °C} \]

Because 1155 is precise to the ones place, and 273.15 is exact, we report the answer for our subtraction to the ones place.

EXERCISE 2.11 - Temperature Conversions

a. N,N-dimethylaniline, C₆H₅N(CH₃)₂, melts at 2.5 °C. What is N,N-dimethylaniline's melting point in °F and K?
b. Benzenethiol, C₆H₅SH, melts at 5.4 °F. What is benzenethiol's melting point in °C and K?
c. The hottest part of the flame on a Bunsen burner is found to be 2.15 × 10³ K. What is this temperature in °C and °F?

Unit Analysis A general technique for doing unit conversions.
Conversion factor A ratio that describes the relationship between two units.
Significant figures The number of meaningful digits in a value. The number of significant figures in a value reflects the value's degree of uncertainty. A larger number of significant figures indicates a smaller degree of uncertainty.
Mass density Mass divided by volume (usually called density).

You can test yourself on the glossary terms at the textbook's Web site.
The goal of this chapter is to teach you to do the following.

1. Define all of the terms in the Chapter Glossary.

Section 2.1 Unit Analysis

2. Write conversion factors that relate the metric base units to units derived from the metric prefixes—for example,

\[
\frac{10^3 \text{ m}}{1 \text{ km}}
\]

3. Use unit analysis to make conversions from one metric unit to another.
4. Write the English-metric conversion factors listed on Table 2.1.
5. Use unit analysis to make conversions between English mass, volume, or length units and metric mass, volume, or length units.

Section 2.2 Rounding Off and Significant Figures

6. Identify each value in a calculation as exact or not exact.
7. Write or identify the number of significant figures in any value that is not exact.
8. Round off answers derived from multiplication and division to the correct number of significant figures.
9. Round off answers derived from calculations involving addition or subtraction to the correct number of decimal positions.

Section 2.3 Density and Density Calculations

10. Provide or recognize the units commonly used to describe the density of solids, liquids, and gases.
11. Use density as a conversion factor to convert between mass and volume.
12. Calculate the density of a substance from its mass and volume.

Section 2.4 Percentage and Percentage Calculations

13. Given a percentage by mass or a percentage by volume, write a conversion factor based on it.
14. Use conversion factors derived from percentages to convert between units of a part and units of the whole.

Section 2.5 A Summary of the Unit Analysis Process

15. Use unit analysis to make unit conversions using conversion factors derived from any relationship that can be described as “something per something”.

Section 2.6 Temperature Conversions

16. Convert a temperature reported in the Celsius, Fahrenheit, or Kelvin scale to both of the other two scales.
Review Questions

1. Write the metric base units and their abbreviations for length, mass, and volume. (See Section 1.4.)

2. Complete the following table by writing the type of measurement the unit represents (mass, length, volume, or temperature), and either the name or the abbreviation for the unit. (See Section 1.4.)

<table>
<thead>
<tr>
<th>Unit</th>
<th>Type of measurement</th>
<th>Abbreviations</th>
<th>Unit</th>
<th>Type of measurement</th>
<th>Abbreviations</th>
</tr>
</thead>
<tbody>
<tr>
<td>milliliter</td>
<td></td>
<td>μg</td>
<td>kilometer</td>
<td></td>
<td>K</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

3. Complete the following relationships between units. (See Section 1.4.)
   a. _____ m = 1 μm
   b. _____ g = 1 Mg
   c. _____ L = 1 mL
   d. _____ m = 1 nm
   e. _____ cm³ = 1 mL
   f. _____ L = 1 m³
   g. _____ kg = 1 t (t = metric ton)
   h. _____ Mg = 1 t (t = metric ton)

4. An empty 2-L graduated cylinder is weighed on a balance and found to have a mass of 1124.2 g. Liquid methanol, CH₃OH, is added to the cylinder, and its volume measured as 1.20 L. The total mass of the methanol and the cylinder is measured as 2073.9 g. Based on the way these data are reported, what do you assume is the range of possible values that each represents? (See Section 1.5.)

Key Ideas

Complete the following statements by writing one of these words or phrases in each blank.

cancel  
correct  
counting  
decimal places  
decrease  
defined  
definitions  
desired  
fewest decimal places  
fewest  
exact  
given value  
grains per cubic centimeter  
grains per liter  
grains per milliliter  
identity  
inexact  
known  
left  
less dense  
mass  
never exact  
one  
part  
“something per something”  
uncertainty  
unit conversion  
unwanted  
variables  
volume  
whole

5. You will find that the stepwise thought process associated with the procedure called unit analysis not only guides you in figuring out how to set up ______________ problems but also gives you confidence that your answers are ______________.
6. The first step in the unit analysis procedure is to identify the unit for the value we want to calculate. We write this on the _____________ side of an equals sign. Next, we identify the _____________ that we will convert into the desired value, and we write it on the other side of the equals sign.

7. In the unit analysis process, we multiply by one or more conversion factors that cancel the _____________ units and generate the _____________ units.

8. Note that the units in a unit analysis setup cancel just like _____________ in an algebraic equation.

9. If you have used correct conversion factors in a unit analysis setup, and if your units _____________ to yield the desired unit or units, you can be confident that you will arrive at the correct answer.

10. Because the English inch is _____________ as 2.54 cm, the number 2.54 in this value is exact.

11. Unless we are told otherwise, we assume that values from measurements have an uncertainty of plus or minus _____________ in the last decimal place reported.

12. If a calculation is performed using all exact values and if the answer is not rounded off, the answer is _____________.

13. When an answer is calculated by multiplying or dividing, we round it off to the same number of significant figures as the _____________ value with the _____________ significant figures.

14. The number of significant figures, which is equal to the number of meaningful digits in a value, reflects the degree of _____________ in the value.

15. Numbers that come from definitions and from _____________ are exact.

16. Values that come from measurements are _____________.

17. When adding or subtracting, round your answer to the same number of _____________ as the inexact value with the _____________.

18. Although there are exceptions, the densities of liquids and solids generally _____________ with increasing temperature.

19. The densities of liquids and solids are usually described in _____________ or _____________.

20. The particles of a gas are much farther apart than the particles of a liquid or solid, so gases are much _____________ than solids and liquids. Thus it is more convenient to describe the densities of gases as _____________.

21. Because the density of a substance depends on the substance’s _____________ and its temperature, it is possible to identify an unknown substance by comparing its density at a particular temperature to the _____________ densities of substances at the same temperature.

22. Because density is reported as a ratio that describes a relationship between two units, the density of a substance can be used in unit analysis to convert between the substance’s _____________ and its _____________.

23. Percentage by mass, the most common form of percentage used in chemical descriptions, is a value that tells us the number of mass units of the _____________ for each 100 mass units of the _____________.

24. Anything that can be read as _____________ can be used as a unit analysis conversion factor.

25. The numbers 1.8, 32, and 273.15 in the equations used for temperature conversions all come from _____________, so they are all exact.
**Problems Relating to Appendix B and Calculator Use.**

If you have not yet read Appendix B, which describes scientific notation, you might want to read it before working the problems that follow. For some of these problems, you might also want to consult your calculator’s instruction manual to determine the most efficient way to complete calculations.

26. Convert the following ordinary decimal numbers to scientific notation.

   a.  67,294  
   b.  438,763,102  
   c.  0.000073  
   d.  0.0000000435

27. Convert the following ordinary decimal numbers to scientific notation.

   a.  1,346.41  
   b.  429,209  
   c.  0.000002056  
   d.  0.00488

28. Convert the following numbers expressed in scientific notation to ordinary decimal numbers.

   a.  $4.097 \times 10^3$  
   b.  $1.55412 \times 10^4$  
   c.  $2.34 \times 10^{-5}$  
   d.  $1.2 \times 10^{-8}$

29. Convert the following numbers expressed in scientific notation to ordinary decimal numbers.

   a.  $6.99723 \times 10^5$  
   b.  $2.333 \times 10^2$  
   c.  $3.775 \times 10^{-3}$  
   d.  $5.1012 \times 10^{-6}$

30. Use your calculator to complete the following calculations.

   a.  $34.25 \times 84.00$  
   b.  $2607 \div 8.25$  
   c.  $425 \div 17 \times 0.22$  
   d.  $(27.001 - 12.866) \div 5.000$

31. Use your calculator to complete the following calculations.

   a.  $36.6 \div 0.0750$  
   b.  $848.8 \times 0.6250$  
   c.  $575.0 \div 5.00 \times 0.20$  
   d.  $2.50 \times (33.141 + 5.099)$

32. Use your calculator to complete the following calculations.

   a.  $10^9 \times 10^3$  
   b.  $10^{12} \div 10^3$  
   c.  $10^3 \times 10^6 \div 10^2$  
   d.  $10^9 \times 10^{-4}$  
   e.  $10^{23} \div 10^{-6}$  
   f.  $10^{-4} \times 10^2 + 10^{-5}$
33. Use your calculator to complete the following calculations. (See your calculator's instruction manual if you need help using a calculator.)

\[ \begin{align*}
\text{a. } & \quad \frac{10^{12}}{10^9} & \quad \text{d. } & \quad \frac{10^{-8}}{10^5} \\
\text{b. } & \quad 10^{14} \times 10^5 & \quad \text{e. } & \quad 10^{-11} \times 10^8 \\
\text{c. } & \quad \frac{10^{17}}{10^3} \times 10^6 & \quad \text{f. } & \quad \frac{10^{17}}{10^{-9}} \times 10^3
\end{align*} \]

34. Use your calculator to complete the following calculations. (See your calculator's instruction manual if you need help using a calculator.)

\[ \begin{align*}
\text{a. } & \quad (9.5 \times 10^5) \times (8.0 \times 10^9) \\
\text{b. } & \quad (6.12 \times 10^{19}) \div (6.00 \times 10^3) \\
\text{c. } & \quad (2.75 \times 10^4) \times (6.00 \times 10^7) + (5.0 \times 10^6) \\
\text{d. } & \quad (8.50 \times 10^{-7}) \times (2.20 \times 10^3) \\
\text{e. } & \quad (8.203 \times 10^9) + 10^{-4} \\
\text{f. } & \quad (7.679 \times 10^{-4} - 3.457 \times 10^{-4}) \div (2.000 \times 10^{-8})
\end{align*} \]

35. Use your calculator to complete the following calculations. (See your calculator's instruction manual if you need help using a calculator.)

\[ \begin{align*}
\text{a. } & \quad (1.206 \times 10^{13}) \div (6.00 \times 10^6) \\
\text{b. } & \quad (5.00 \times 10^{23}) \times (4.4 \times 10^{17}) \\
\text{c. } & \quad (7.500 \times 10^3) \times (3.500 \times 10^9) + (2.50 \times 10^{15}) \\
\text{d. } & \quad (1.85 \times 10^4) \times (2.0 \times 10^{-12}) \\
\text{e. } & \quad (1.809 \times 10^{-9}) \div (9.00 \times 10^{-12}) \\
\text{f. } & \quad (7.131 \times 10^6 - 4.006 \times 10^6) \div 10^{-12}
\end{align*} \]

**Section 2.1 Unit Analysis**

36. Complete each of the following conversion factors by filling in the blank on the top of the ratio.

\[ \begin{align*}
a. & \quad \frac{\text{___ m}}{1 \text{ km}} \\
b. & \quad \frac{\text{___ cm}}{1 \text{ m}} \\
c. & \quad \frac{\text{___ mm}}{1 \text{ m}} \\
d. & \quad \frac{\text{___ cm}^3}{1 \text{ mL}} \\
e. & \quad \frac{\text{___ cm}}{1 \text{ in.}} \\
f. & \quad \frac{\text{___ g}}{1 \text{ lb}}
\end{align*} \]

**Objective 2** **Objective 4**
37. Complete each of the following conversion factors by filling in the blank on the top of the ratio.

- a. \( \frac{____ \mu m}{1 \text{ m}} \)
- b. \( \frac{____ \text{ nm}}{1 \text{ m}} \)
- c. \( \frac{____ \text{ kg}}{1 \text{ metric ton}} \)
- d. \( \frac{____ \text{ L}}{1 \text{ gal}} \)
- e. \( \frac{____ \text{ km}}{1 \text{ mi}} \)
- f. \( \frac{____ \text{ in.}}{1 \text{ m}} \)

38. Complete each of the following conversion factors by filling in the blank on the top of the ratio.

- a. \( \frac{____ \text{ g}}{1 \text{ kg}} \)
- b. \( \frac{____ \text{ mg}}{1 \text{ g}} \)
- c. \( \frac{____ \text{ yd}}{1 \text{ m}} \)
- d. \( \frac{____ \text{ lb}}{1 \text{ kg}} \)

39. Complete each of the following conversion factors by filling in the blank on the top of the ratio.

- a. \( \frac{____ \mu g}{1 \text{ g}} \)
- b. \( \frac{____ \text{ mL}}{1 \text{ L}} \)
- c. \( \frac{____ \mu L}{1 \text{ L}} \)
- d. \( \frac{____ \text{ g}}{1 \text{ oz}} \)
- e. \( \frac{____ \text{ qt}}{1 \text{ L}} \)

40. The mass of an electron is \( 9.1093897 \times 10^{-31} \text{ kg} \). What is this mass in grams?

41. The diameter of a human hair is 2.5 micrometers. What is this diameter in meters?

42. The diameter of typical bacteria cells is 0.00032 centimeters. What is this diameter in micrometers?

43. The mass of a proton is \( 1.6726231 \times 10^{-27} \text{ kg} \). What is this mass in micrograms?

44. The thyroid gland is the largest of the endocrine glands, with a mass between 20 and 25 grams. What is the mass in pounds of a thyroid gland measuring 22.456 grams?

45. The average human body contains 5.2 liters of blood. What is this volume in gallons?

46. The mass of a neutron is \( 1.674929 \times 10^{-27} \text{ kg} \). Convert this to ounces. (There are 16 oz/lb.)

47. The earth weighs about \( 1 \times 10^{21} \text{ tons} \). Convert this to gigagrams. (There are 2000 lb/ton.)

48. A red blood cell is \( 8.7 \times 10^{-5} \text{ inches} \) thick. What is this thickness in micrometers?
49. The gallbladder has a capacity of between 1.2 and 1.7 fluid ounces. What is the capacity in milliliters of a gallbladder that can hold 1.42 fluid ounces? (There are 32 fl oz/qt.)

Section 2.2 Rounding Off and Significant Figures

50. Decide whether each of the numbers shown in bold type below is exact or not. If it is not exact, write the number of significant figures in it.
   a. The approximate volume of the ocean, $1.5 \times 10^{21}$ L.
   b. A count of 24 instructors in the physical science division of a state college.
   c. The 54% of the instructors in the physical science division who are women (determined by counting 13 women in the total of 24 instructors and then calculating the percentage)
   d. The 25% of the instructors in the physical science division who are left handed (determined by counting 6 left handed instructors in the total of 24 and then calculating the percentage)
   e. $\frac{16 \text{ oz}}{1 \text{ lb}}$
   f. $\frac{10^6 \text{ mcm}}{1 \text{ m}}$
   g. $\frac{1.057 \text{ qt}}{1 \text{ L}}$
   h. A measurement of 107.200 g water
   i. A mass of 0.2363 lb water (calculated from Part h, using $\frac{453.6 \text{ g}}{1 \text{ lb}}$ as a conversion factor)
   j. A mass of $1.182 \times 10^{-4}$ tons (calculated from the 0.2363 lb of the water described in Part i.)

51. Decide whether each of the numbers shown in bold type below is exact or not. If it is not exact, write the number of significant figures in it.
   a. $\frac{10^9 \text{ ng}}{1 \text{ g}}$
   b. $\frac{32 \text{ fl oz}}{1 \text{ qt}}$
   c. $\frac{1.094 \text{ yd}}{1 \text{ m}}$
   d. The diameter of the moon, $3.480 \times 10^3$ km
   e. A measured volume of 8.0 mL water
   f. A volume 0.0080 L water, calculated from the volume in Part e, using $\frac{1 \text{ L}}{10^3 \text{ mL}}$
   g. A volume of 0.0085 qt water, calculated from the volume in Part f, using $\frac{1.057 \text{ qt}}{1 \text{ L}}$
   h. The count of 114 symbols for elements on a periodic table
   i. The 40% of halogens that are gases at normal room temperature and pressure (determined by counting 2 gaseous halogens out of the total of 5 halogens and then calculating the percentage)
   j. The 9.6% of the known elements that are gases at normal room temperature and pressure (determined by counting 11 gaseous elements out of the 114 elements total and then calculating the percentage)
52. Assuming that the following numbers are not exact, how many significant figures does each number have?
   a. 13.811  
   b. 0.0445  
   c. 505  
   d. 9.5004  
   e. 81.00

53. Assuming that the following numbers are not exact, how many significant figures does each number have?
   a. 9,875  
   b. 102.405  
   c. 10.000  
   d. 0.00012  
   e. 0.411

54. Assuming that the following numbers are not exact, how many significant figures does each number have?
   a. \(4.75 \times 10^{23}\)  
   b. \(3.009 \times 10^{-3}\)  
   c. \(4.000 \times 10^{13}\)

55. Assuming that the following numbers are not exact, how many significant figures does each number have?
   a. \(2.00 \times 10^{8}\)  
   b. \(1.998 \times 10^{-7}\)  
   c. \(2.0045 \times 10^{-5}\)

56. Convert each of the following numbers to a number having 3 significant figures.
   a. 34.579  
   b. 193.405  
   c. 23.995  
   d. 0.003882  
   e. 0.023  
   f. 2.846.5  
   g. \(7.8354 \times 10^{4}\)

57. Convert each of the following numbers to a number having 4 significant figures.
   a. 4.30398  
   b. 0.000421  
   c. \(4.44802 \times 10^{-19}\)  
   d. 99.9975  
   e. 11,687.42  
   f. 874.992
58. Complete the following calculations and report your answers with the correct number of significant figures. The exponential factors, such as 10^3, are exact, and the 2.54 in part (c) is exact. All the other numbers are not exact.

a. \[
\frac{2.45 \times 10^{-5}(10^{12})}{(10^3)237.00} =
\]

b. \[
\frac{16.050(10^3)}{(24.8 - 19.4)(1.057)(453.6)} =
\]

c. \[
\frac{4.77 \times 10^{11}(2.54)^2(73.00)}{(10^3)} =
\]

59. Complete the following calculations and report your answers with the correct number of significant figures. The exponential factors, such as 10^3, are exact, and the 5280 in part (c) is exact. All the other numbers are not exact.

a. \[
\frac{8.9932 \times 10^{-2}(10^3)0.0048}{(10^{-6})7.140} =
\]

b. \[
\frac{(44.945 - 23.775)(10^3)3.785412}{(15.200)(453.59237)} =
\]

c. \[
\frac{456.8(5280)^2}{(10^3)^2(1.609)^2} =
\]

60. Report the answers to the following calculations to the correct number of decimal positions. Assume that each number is precise to ±1 in the last decimal position reported.

a. \[0.8995 + 99.24 = \]

b. \[88 - 87.3 = \]

61. Report the answers to the following calculations to the correct number of decimal positions. Assume that each number is precise to ±1 in the last decimal position reported.

a. \[23.40 - 18.2 = \]

b. \[948.75 + 62.45 = \]

Section 2.3 Density and Density Calculations

Because the ability to make unit conversions using the unit analysis format is an extremely important skill, be sure to set up each of the following calculations using the unit analysis format, even if you see another way to work the problem, and even if another technique seems easier.

62. A piece of balsa wood has a mass of 15.196 g and a volume of 0.1266 L. What is its density in g/mL?

63. A ball of clay has a mass of 2.65 lb and a volume of 0.5025 qt. What is its density in g/mL?

64. The density of water at 0 °C is 0.99987 g/mL. What is the mass in kilograms of 185.0 mL of water?

65. The density of water at 3.98 °C is 1.00000 g/mL. What is the mass in pounds of 16.785 L of water?
66. The density of a piece of ebony wood is 1.174 g/mL. What is the volume in quarts of a 2.1549 lb piece of this ebony wood?

67. The density of whole blood is 1.05 g/mL. A typical adult has about 5.5 L of whole blood. What is the mass in pounds of this amount of whole blood?

Section 2.4 Percentage and Percentage Calculations

68. The mass of the ocean is about $1.8 \times 10^{21}$ kg. If the ocean contains 1.076% by mass sodium ions, $\text{Na}^+$, what is the mass in kilograms of $\text{Na}^+$ in the ocean?

69. While you are at rest, your brain gets about 15% by volume of your blood. If your body contains 5.2 L of blood, how many liters of blood are in your brain at rest?...how many quarts?

70. While you are doing heavy work, your heart pumps up to 25.0 L of blood per minute. Your brain gets about 3-4% by volume of your blood under these conditions. What volume of blood in liters is pumped through your brain in 125 minutes of work that causes your heart to pump 22.0 L per minute, 3.43% of which goes to your brain?

71. While you are doing heavy work, your heart pumps up to 25.0 L of blood per minute. Your muscles get about 80% by volume of your blood under these conditions. What volume of blood in quarts is pumped through your muscles in 105 minutes of work that causes your heart to pump 21.0 L per minute, 79.25% by volume of which goes to your muscles?

72. In chemical reactions that release energy, from $10^{-8}$% to $10^{-7}$% of the mass of the reacting substances is converted to energy. Consider a chemical reaction for which $1.8 \times 10^{-8}$% of the mass is converted into energy. What mass in milligrams is converted into energy when 1.0 $\times 10^{3}$ kilograms of substance reacts?

73. In nuclear fusion, about 0.60% of the mass of the fusing substances is converted to energy. What mass in grams is converted into energy when 22 kilograms of substance undergoes fusion?

Section 2.5 A Summary of the Unit Analysis Process

74. If an elevator moves 1340 ft to the 103rd floor of the Sears Tower in Chicago in 45 seconds, what is the velocity (distance traveled divided by time) of the elevator in kilometers per hour?

75. The moon orbits the sun with a velocity of $2.2 \times 10^{4}$ miles per hour. What is this velocity in meters per second?

76. Sound travels at a velocity of 333 m/s. How long does it take for sound to travel the length of a 100-yard football field?

77. How many miles can a commercial jetliner flying at 253 meters per second travel in 6.0 hours?
Chapter Problems

78. A peanut butter sandwich provides about $1.4 \times 10^3$ kJ of energy. A typical adult uses about 95 kcal/hr of energy while sitting. If all of the energy in one peanut butter sandwich were to be burned off by sitting, how many hours would it be before this energy was used? (A kcal is a dietary calorie. There are 4.184 J/cal.)

79. One-third cup of vanilla ice cream provides about 145 kcal of energy. A typical adult uses about 195 kcal/hr of energy while walking. If all of the energy in one-third of a cup of vanilla ice cream were to be burned off by walking, how many minutes would it take for this energy to be used? (A kcal is a dietary calorie.)

80. When one gram of hydrogen gas, $H_2(g)$, is burned, 141.8 kJ of heat are released. How much heat is released when 2.3456 kg of hydrogen gas are burned?

81. When one gram of liquid ethanol, $C_2H_5OH(l)$, is burned, 29.7 kJ of heat are released. How much heat is released when 4.274 pounds of liquid ethanol are burned?

82. When one gram of carbon in the graphite form is burned, 32.8 kJ of heat are released. How many kilograms of graphite must be burned to release $1.456 \times 10^4$ kJ of heat?

83. When one gram of methane gas, $CH_4(g)$, is burned, 55.5 kJ of heat are released. How many pounds of methane gas must be burned to release $2.578 \times 10^3$ kJ of heat?

84. The average adult male needs about 58 g of protein in the diet each day. A can of vegetarian refried beans has 6.0 g of protein per serving. Each serving is 128 g of beans. If your only dietary source of protein were vegetarian refried beans, how many pounds of beans would you need to eat each day?

85. The average adult needs at least $1.50 \times 10^2$ g of carbohydrates in the diet each day. A can of vegetarian refried beans has 19 g of carbohydrate per serving. Each serving is 128 g of beans. If your only dietary source of carbohydrate were vegetarian refried beans, how many pounds of beans would you need to eat each day?

86. About $6.0 \times 10^5$ tons of 30% by mass hydrochloric acid, $HCl(aq)$, are used to remove metal oxides from metals to prepare them for painting or for the addition of a chrome covering. How many kilograms of pure HCl would be used to make this hydrochloric acid? (Assume that 30% has two significant figures. There are 2000 lb/ton.)

87. Normal glucose levels in the blood are from 70 to 110 mg glucose per 100 mL of blood. If the level falls too low, there can be brain damage. If a person has a glucose level of 108 mg/100 mL, what is the total mass of glucose in grams in 5.10 L of blood?

88. A typical non-obese male has about 11 kg of fat. Each gram of fat can provide the body with about 38 kJ of energy. If this person requires $8.0 \times 10^3$ kJ of energy per day to survive, how many days could he survive on his fat alone?

89. The kidneys of a normal adult male filter 125 mL of blood per minute. How many gallons of blood are filtered in one day?

90. During quiet breathing, a person breathes in about 6 L of air per minute. If a person breathes in an average of 6.814 L of air per minute, what volume of air in liters does this person breathe in 1 day?
91. During exercise, a person breathes in between 100 and 200 L of air per minute. If a person is exercising enough to breathe in an average of 125.6 L of air per minute, what total volume of air in liters is breathed in exactly one hour of exercise?

92. The kidneys of a normal adult female filter 115 mL of blood per minute. If this person has 5.345 quarts of blood, how many minutes will it take to filter all of her blood once?

93. A normal hemoglobin concentration in the blood is 15 g/100 mL of blood. How many kilograms of hemoglobin are there in a person who has 5.5 L of blood?

94. We lose between 0.2 and 1 liter of water from our skin and sweat glands each day. For a person who loses an average of 0.89 L H₂O per day in this manner, how many quarts of water are lost from the skin and sweat glands in 30 days?

95. Normal blood contains from 3.3 to 5.1 mg of amino acids per 100 mL of blood. If a person has 5.33 L of blood and 4.784 mg of amino acids per 100 mL of blood, how many grams of amino acids does the blood contain?

96. The average heart rate is 75 beats/min. How many times does the average person’s heart beat in a week?

97. The average heart rate is 75 beats/min. Each beat pumps about 75 mL of blood. How many liters of blood does the average person’s heart pump in a week?

98. In optimum conditions, one molecule of the enzyme carbonic anhydrase can convert \(3.6 \times 10^5\) molecules per minute of carbonic acid, \(H₂CO₃\), to carbon dioxide, \(CO₂\), and water, \(H₂O\). How many molecules could be converted by one of these enzyme molecules in one week?

99. In optimum conditions, one molecule of the enzyme fumarase can convert \(8 \times 10^2\) molecules per minute of fumarate to malate. How many molecules could be converted by one of these enzyme molecules in 30 days?

100. In optimum conditions, one molecule of the enzyme amylase can convert \(1.0 \times 10^5\) molecules per minute of starch to the sugar maltose. How many days would it take one of these enzyme molecules to convert a billion \((1.0 \times 10^9)\) starch molecules?

101. There are about \(1 \times 10^5\) chemical reactions per second in the 10 billion nerve cells in the brain. How many chemical reactions take place in a day in a single nerve cell?

102. When you sneeze, you close your eyes for about 1.00 s. If you are driving 65 miles per hour on the freeway and you sneeze, how many feet do you travel with your eyes closed?
Section 2.6 Temperature Conversions

103. Butter melts at 31 °C. What is this temperature in °F? ….in K?

104. Dry ice freezes at −79 °C. What is this temperature in °F? ….in K?

105. A saturated salt solution boils at 226 °F. What is this temperature in °C? ….in K?

106. Table salt, sodium chloride, melts at 801 °C. What is this temperature in °F? ….in K?

107. Iron boils at 3023 K. What is this temperature in °C? ….in °F?

108. Absolute zero, the lowest possible temperature, is exactly 0 K. What is this temperature in °C?….in °F?

109. The surface of the sun is $1.0 \times 10^4$ °F. What is this temperature in °C? …in K?